Published by Institute of Physics Publishing for SISSA
Received: *April 11, 2005*
Accepted: *May 23, 2005*
Published: *June 16, 2005*

$\mathcal{N} = 1$ effective potential from dual type-IIA D6/O6 orientifolds with general fluxes

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Abstract: We consider $\mathcal{N} = 1$ compactifications of the type-IIA theory on the $T^6/(Z_2 \times Z_2)$ orbifold and O6 orientifold, in the presence of D6-branes and general NSNS, RR and Scherk-Schwarz geometrical fluxes. Introducing a suitable dual formulation of the theory, we derive and solve the Bianchi identities, and show how certain combinations of fluxes can relax the constraints on D6-brane configurations coming from the cancellation of RR tadpoles. We then compute, via generalized dimensional reduction, the $\mathcal{N} = 1$, $D = 4$ effective potential for the seven main moduli, and comment on the relation with truncated $\mathcal{N} = 4$ gaugings. As a byproduct, we obtain a general geometrical expression for the superpotential. We finally identify a family of fluxes, compatible with all Bianchi identities, that perturbatively stabilize all seven moduli in supersymmetric AdS_4.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, Supergravity Models, Compactification and String Models.

JHEP06(2005)047

Contents

1. Introduction

Recently, there has been a lot of interest in orientifold compactifications of type-II theories, with exact or spontaneously broken $\mathcal{N} = 1$ supersymmetry, in the presence of branes and fluxes (for reviews and references, see e.g. [1]). The reason is twofold. On the one hand, new possibilities appear for constructing semi-realistic models, with the Standard Model fields emerging from fluctuations of intersecting and/or magnetized branes. On the other hand, the possibility of additional RR fluxes with respect to the heterotic theory provides new opportunities for perturbatively addressing unsolved problems such as vacuum selection, supersymmetry breaking and moduli stabilization. The ultimate goal of such a program would be to obtain explicit, consistent constructions of flux vacua with broken supersymmetry, an approximately flat four-dimensional space and all moduli stabilized, with D-branes reproducing the gauge structure and chiral matter of the Standard Model. We are still far from this goal, but some first encouraging steps have been made.

In heterotic compactifications (for a review and references, see e.g. [2]), and in the known IIB compactifications [1], the available perturbative fluxes in the closed string sector are not sufficient to produce $\mathcal{N} = 1$ supersymmetric AdS_4 vacua and to fix all the geometrical moduli. On the other hand, for type-IIA compactifications examples exist of stable AdS_4 vacua, obtained via perturbative fluxes only [3 – 5].

The authors of [3] considered IIA compactifications on suitable manifolds with non-trivial torsion and SU(3) or SU(2) structures, to get flux vacua with unbroken $\mathcal{N} = 1$ supersymmetry. In particular, they found an AdS_4 supersymmetric vacuum on a nearly-Kähler manifold when all NSNS and RR fluxes are turned on. Further investigations along similar lines were carried out in [5].

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The authors of [4] focused on the D6/O6 orientifold with the $T^6/(Z_2 \times Z_2)$ orbifold, which combines a simple geometry of the internal space with a very rich structure of fluxes. Among the possibilities allowed by the general system of NSNS, RR and geometrical Scherk-Schwarz fluxes, they got a stable AdS_4 supersymmetric vacuum with all seven geometrical moduli fixed, as well as non-supersymmetric vacua corresponding to no-scale models or runaway potentials.

O6 compactifications on the $T^6/(Z_2 \times Z_2)$ orbifold have also a phenomenological interest, since semi-realistic chiral $\mathcal{N} = 1$ models with intersecting D6-branes have been formulated in such a framework [6]. It is then interesting to investigate the relation between the compactified $D = 10$ theory and the resulting effective $D = 4$ theory in more detail: this is the goal of the present paper. For simplicity, we will set to zero all the brane excitations and the associated magnetic fluxes: such a setup is obviously insufficient to give a realistic model, but is a good training ground for a quantitative and self-consistent discussion of vacuum selection, supersymmetry breaking and moduli stabilization.

Since our work complements and extends the work of [4], we first summarize the results of the latter. That paper focused on the O6 orientifold of the type-IIA theory (which would give rise, on the six-torus T^6, to an effective $N = 4$, $D = 4$ supergravity), consistently combined with the $T^6/(Z_2 \times Z_2)$ orbifold, to preserve an exact or spontaneously broken $\mathcal{N} = 1$, $D = 4$ supergravity. First, it classified the most general fluxes in the closed string sector that are invariant under the combined orbifold and orientifold projections: they include fluxes of the NSNS 3-form H; fluxes of the even RR forms $G^{(0)}$ (corresponding to the mass parameter of massive IIA supergravity [7]), $G^{(2)}$, $G^{(4)}$, $G^{(6)}$; also, geometrical fluxes ω, associated with the internal components of the spin connection, and corresponding to Scherk-Schwarz [8] or, equivalently, twisted tori compactifications. Then, it focused the attention (as we do in the present paper, leaving further generalizations to future work) on the effective theory for the seven main moduli coming from the closed string sector. In particular, it determined the general form of the effective Kähler potential and superpotential, after identifying a suitable field basis. Instead of considering the Bianchi identities and the RR tadpole cancellation conditions of the compactified $D = 10$ theory, it explored the constraints on the $\mathcal{N} = 1$ superpotential corresponding to consistent gaugings of the underlying $\mathcal{N} = 4$, $D = 4$ effective supergravity. For toroidal heterotic compactifications, this was proved to be a successful strategy, since the Bianchi identities of the reduced theory, in the presence of general fluxes, are exactly equivalent [9] to the conditions for a $\mathcal{N} = 4$ gauging. This is also true in several M-theory and type-II examples, as recently discussed in [10].

The goal of the present paper is to consider the same class of IIA compactifications as in [4], investigating the structure of the $D = 4$ effective theory for the seven main moduli directly by generalized $\mathcal{N} = 1$ dimensional reduction, rather than from the point of view of a truncated gauged $\mathcal{N} = 4$ supergravity. To deal with general systems of fluxes, D6-branes and O6-planes, we will introduce a dual formulation of the IIA theory. A similar approach was followed in [11], without geometrical fluxes, and in [12] for compactifications to two dimensions with RR fluxes only. Our method will allow us to consider, beyond the limits of [4], consistent $\mathcal{N} = 1$ compactifications that cannot be viewed as truncations

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of $\mathcal{N}=4$ supergravities. It will also allow us to give a geometrical interpretation to the effective superpotential, and to discuss the subtle interplay between the Bianchi identities of the compactified $D=10$ theory and the conditions for a consistent gauging of the effective $\mathcal{N}=4$ theory, in the cases where the $\mathcal{N}=1$ effective theory can be obtained by a truncation of the former.

Our paper is organized as follows.

In section 2, we define the class of IIA compactifications under consideration, and, after fixing the conventions and recalling the standard form of (massive) IIA supergravity, we write down the bulk and brane actions in the dual version.

In section 3, we derive the Bianchi identities in the presence of general fluxes (NSNS, RR, geometrical) and of D6/O6 systems compatible with $\mathcal{N}=1$ supersymmetry. Their general solution has the form

$$\begin{aligned} H &= dB + \omega\ B + \overline{H}\,, \\ G^{(p)} &= dC^{(p-1)} + \omega\ C^{(p-1)} + H \wedge C^{(p-3)} + (\overline{\mathbf{G}}\ e^{-B})^{(p)}\,, \end{aligned} \tag{1.1}$$

for NSNS and RR fields respectively, with ω, $\overline{H}$ and $\overline{G}^{(p)}$ constant fluxes subjected to the integrability conditions:

$$\begin{aligned} \omega\ \omega = 0\,, \qquad \omega\ \overline{H} = 0\,, \\ \omega\ \overline{G}^{(p)} + \overline{H} \wedge \overline{G}^{(p-2)} = 0\,. \end{aligned} \tag{1.2}$$

D6-brane and O6-plane contributions only show up in the non-trivial Bianchi identities for the *phantom* components of $G^{(2)}$ that are projected out by the orbifold:

$$\frac{1}{2}\left(dG^{(2)} + \omega\, G^{(2)} + HG^{(0)}\right) = \sum_a^{D6,O6} \mu_a \delta^{(3)}(\pi_a)\,. \tag{1.3}$$

Their integrability conditions provide additional constraints for fluxes and localized charges, corresponding to modified RR tadpole cancellation conditions.

In section 4, we derive the $\mathcal{N}=1$, $D=4$ effective potential via generalized dimensional reduction, and check that it agrees with the general form of the effective Kähler potential and superpotential, obtained in [4] via consistent truncation of an underlying $\mathcal{N}=4$, $D=4$ gauged supergravity. Our derivation allows to rewrite the generic $\mathcal{N}=1$ effective superpotential in a suggestive geometrical form, which can be explicitly checked on our example and generalizes those previously derived (or conjectured on the basis of duality) by various authors [12 − 14]:

$$W = \frac{1}{4}\int_{\mathcal{M}_6} \overline{\mathbf{G}}\, e^{iJ^c} - i\left(\overline{H} - i\omega\, J^c\right) \wedge \Omega^c\,, \tag{1.4}$$

where J^c and Ω^c are the complexified Kähler 2-form and the holomorphic 3-form, respectively. We also find that the Bianchi identities, including those for the geometrical fluxes alone, can be satisfied by systems of fluxes and branes that do not correspond to $\mathcal{N}=4$, $D=4$ gaugings, thereby relaxing some of the constraints on the parameters of the $\mathcal{N}=1$ superpotential.

In section 5 we produce examples of consistent systems of fluxes, satisfying all the Bianchi identities (including those for the geometrical fluxes), that give rise to stable supersymmetric AdS_4 vacua with all seven geometrical moduli fixed. In these examples, those corresponding to $\mathcal{N}=4$ gaugings involve all types of allowed fluxes. If we ask only for the Bianchi identities, without requiring an underlying gauged $\mathcal{N}=4$ supergravity, then the allowed fluxes that stabilize all seven main moduli depend on more parameters, and stabilization can be achieved even if some types of fluxes are switched off.

In the final section we briefly summarize our results and comment on the prospects for extending the present work.

2. IIA theory with D6/O6 on $T^6/(Z_2 \times Z_2)$ and its dual formulation

The bosonic sector of $D=10$ type-IIA supergravity consists of the following fields: in the NSNS sector, the (string-frame) metric g_{MN}, the 2-form potential[1] B and the dilaton Φ; in the RR sector, the $(2k+1)$-form potentials $C^{(2k+1)}$, $(k=0,\dots,4)$. Here and in the following, we stick to the conventions of [2] unless otherwise stated, and we split the $D=10$ space-time indices as $M=[\mu=0,\dots,3\,;\,i=5,\dots,10]$. Actually, the degrees of freedom of the RR potentials are not all independent, being related by Poincaré duality. The corresponding action can then be expressed as a function of the independent fields only, usually identified with $C^{(1)}$ and $C^{(3)}$. This leads to the standard formulation of $D=10$ IIA supergravity, whose action reads, in a schematic but self-explanatory notation:

$$
\begin{aligned}
S_{IIA} &= S_{NS}+S_R+S_{CS}\,, && (2.1)\\
S_{NS} &= \int d^{10}x\, e_{10}\, \frac{e^{-2\Phi}}{2}\left[R_{10}+4\,(\partial\Phi)^2-\frac{1}{2}\,(H)^2\right], && (2.2)\\
S_R &= -\frac{1}{4}\int d^{10}x\, e_{10}\left[M^2+(F^{(2)})^2+(\widetilde{F}^{(4)})^2\right], && (2.3)\\
S_{CS} &= -\frac{1}{4}\int d^{10}x\, B\wedge F^{(4)}\wedge F^{(4)}\,, && (2.4)
\end{aligned}
$$

where

$$
\begin{aligned}
H &= dB\,,\\
F^{(2)} &= dC^{(1)}+M\,B\,,\\
F^{(4)} &= dC^{(3)}+\frac{1}{2}\,M\,B\wedge B\,,\\
\widetilde{F}^{(4)} &= F^{(4)}-C^{(1)}\wedge H\,, \qquad (2.5)
\end{aligned}
$$

and $M\equiv F^{(0)}$ is the mass parameter of massive IIA supergravity [7].

In the presence of localized objects, the standard formulation of (massive) IIA supergravity is not the optimal choice, since D$(p-1)$-branes carry RR charges and naturally (i.e. electrically) couple to p-form potentials. Thus, depending on the type of D$(p-1)$-branes considered in the effective theory, it is more convenient to use a suitably dualized action that includes these fields, along the lines of reference [11].

[1]Since there is no risk of confusion, we do not attach a "(2)" to B and a "(3)" to its field-strength H.

The general procedure for the construction of the dual action works as follows. The first step consists in choosing the independent degrees of freedom: for each couple of p-form potentials $C^{(p)}$ $(p = n, 8-n)$, a single p-form potential and its dual field strength $G^{(9-p)}$ have to be chosen. Both these fields are treated as independent, with the potential entering the action with the role of a Lagrange multiplier. The second step is to construct the action which reproduces the standard formulation of (2.1) in the "no-flux" limit, after elimination of the Lagrange multipliers. The equations of motion for the RR potentials actually give the Bianchi identities (BI in the following) for their dual field strengths. The solutions of these BI give then the explicit expressions for the $(9-p)$-form field strengths, in terms of a $(8-p)$-form potential and a combination of fluxes and other fields.

In this work we are interested in the representative case of type-IIA supergravity with O6-planes and D6-branes, compactified on the orbifold $T^6/(Z_2 \times Z_2)$ in the presence of general fluxes. However, as will be clear in the following, most of our results have more general validity.

We start by defining the action of the orbifold projection on the internal coordinates x^i $(i = 5, \ldots, 10)$ of the factorized six-torus $T^6 = T^2 \times T^2 \times T^2$ as

$$Z_2 \;:\; (z^1, z^2, z^3) \to (-z^1, -z^2, z^3)\,, \qquad Z_2' \;:\; (z^1, z^2, z^3) \to (z^1, -z^2, -z^3)\,, \tag{2.6}$$

where

$$z^1 = x^5 + i\,x^6\,, \qquad z^2 = x^7 + i\,x^8\,, \qquad z^3 = x^9 + i\,x^{10}\,, \tag{2.7}$$

and the action of the orientifold involution as

$$\mathcal{R} \;:\; (z^1, z^2, z^3) \to (-\overline{z}^1, -\overline{z}^2, -\overline{z}^3)\,. \tag{2.8}$$

The minus sign in the above equation has been chosen to match the conventions of [4]. The action of $\mathcal{R}$ is compatible with the presence of O6-planes with internal coordinates $(6, 8, 10)$. The action of the orbifold group produces extra O6-planes in the three directions $[(6, 7, 9), (5, 8, 9), (5, 7, 10)]$. The intrinsic parities of the fields with respect to the orientifold projection are: $+1$ for the g_{MN}, Φ and the RR p-form potentials with $p = 3, 7$; -1 for B and the RR p-form potentials with $p = 1, 5, 9$.

The bulk scalar fields surviving the orbifold and orientifold projections are: the dilaton Φ; three Kähler moduli t_A $(A = 1, 2, 3)$ and three complex structure moduli u_A from the diagonal components g_{ii} $(i = 5, \ldots, 10)$ of the internal metric; three axions τ_A from the NSNS 2-form potentials[2] $B_{56|\cdot|\cdot}$; one plus three axions σ and ν_A from the RR 3-form potentials $C^{(3)}_{6810|\cdot|\cdot|\cdot}$.

In the case under consideration, the dualization in the RR sector does not affect the NSNS part of the action, which remains the same as in (2.2). For the part of the action involving the RR sector we adopt a dual formulation along the lines of [11]. As we will see,

[2]In our short-hand notation, $X_{I|\cdot|\cdot|\ldots}$, where X is a generic field and I is a group of space-time indices, means the component X_I and all the other components with the same number of indices as I and the same parity under the orbifold and orientifold projections. For example, $B_{56|\cdot|\cdot} \equiv B_{56}, B_{78}, B_{910}$, and $C^{(3)}_{6810|\cdot|\cdot|\cdot} \equiv C_{6810}, C_{679}, C_{789}, C_{5710}$.

$C^{(-1)} \leftrightarrow C^{(9)}$	$C^{(1)} \leftrightarrow C^{(7)}$		$C^{(3)} \leftrightarrow C^{(5)}$		
$\times$	$\times$	$\times$	$\times$	$C^{(3)}_{6810\vert\cdot\vert\cdot\vert\cdot}$	$C^{(3)}_{\mu\nu\rho}$
$G^{(0)}$	$G^{(2)}_{56\vert\cdot\vert\cdot}$	$(\times)$	$G^{(4)}_{5678\vert\cdot\vert\cdot}$	$G^{(4)}_{\mu\{6810\vert\cdot\vert\cdot\vert\cdot\}}$	$G^{(4)}_{\mu\nu\rho\sigma}$
$G^{(10)}$	$G^{(8)}_{\mu\nu\rho\sigma\{5678\vert\cdot\vert\cdot\}}$	$\times$	$G^{(6)}_{\mu\nu\rho\sigma\{56\vert\cdot\vert\cdot\}}$	$G^{(6)}_{\mu\nu\rho\{579\vert\cdot\vert\cdot\vert\cdot\}}$	$G^{(6)}_{5678910}$
$C^{(9)}_{\mu\nu\rho\{5678910\}}$	$C^{(7)}_{\mu\nu\rho\{5678\vert\cdot\vert\cdot\}}$	$C^{(7)}_{\mu\nu\rho\sigma\{6810\vert\cdot\vert\cdot\vert\cdot\}}$	$C^{(5)}_{\mu\nu\rho\{56\vert\cdot\vert\cdot\}}$	$C^{(5)}_{\mu\nu\{579\vert\cdot\vert\cdot\vert\cdot\}}$	$\times$

Table 1: IIA bulk fields surviving the orbifold and orientifold projections. Each column may contain the invariant components of p-form potentials, the corresponding $(p+1)$-form field strengths and their duals. Each dot indicates a different combination of indices compatible with the projections.

this formalism will allow us to consistently derive both the effective action in the presence of D6-branes and general fluxes (NSNS, RR, geometrical), and the BI for the RR field strengths.

For our purposes, we need only the action for the components of the fields that survive the orbifold and orientifold projections. The complete list of these fields is given in table 1, from which we can see that the choice of independent fields is almost completely fixed by the projections. The only components where a choice is allowed are in fact $C^{(3)}_{6810|\cdot|\cdot|\cdot}$ and $C^{(5)}_{\mu\nu\{579|\cdot|\cdot|\cdot\}}$. For these components, taking as Lagrange multiplier either $C^{(3)}$ or $C^{(5)}$ is equivalent to arranging the corresponding $D=4$ axionic fields into either a linear or a chiral multiplet, respectively. We choose the second option, which makes contact with the standard formulation of the effective $\mathcal{N}=1$, $D=4$ supergravity.

Notice also that, at variance with the cases discussed in [11], for the dual couple $(C^{(5)}, C^{(3)})$ we must choose different dualizations for different components, consistently with the action of the orbifold and orientifold projections on the space-time coordinates.

Finally, there is a subtlety with the components $C^{(7)}_{\mu\nu\rho\sigma\ \{6810|\cdot|\cdot|\cdot\}}$, whose dual field strengths $G^{(2)}$ are truncated away by the orientifold and orbifold projections. Although these fields cannot contribute to the effective scalar potential, their derivatives can survive the projections and, as we will show, they do indeed lead to non-trivial BI.

Summarizing, the couples of independent bulk fields that we keep in the action are

$$\begin{aligned}
&G^{(0)} \leftrightarrow C^{(9)}_{\mu\nu\rho\{5678910\}}\,, \\
&G^{(2)}_{56|\cdot|\cdot} \leftrightarrow C^{(7)}_{\mu\nu\rho\{78910|\cdot|\cdot\}}\,, \qquad G^{(2)} \leftrightarrow C^{(7)}_{\mu\nu\rho\sigma\{6810|\cdot|\cdot|\cdot\}}\,, \\
&G^{(4)}_{5678|\cdot|\cdot} \leftrightarrow C^{(5)}_{\mu\nu\rho\{910|\cdot|\cdot\}}\,, \qquad G^{(4)}_{\mu\{6810|\cdot|\cdot|\cdot\}} \leftrightarrow C^{(5)}_{\mu\nu\{579|\cdot|\cdot|\cdot\}}\,, \\
&G^{(6)}_{5678910} \leftrightarrow C^{(3)}_{\mu\nu\rho}\,.
\end{aligned} \tag{2.9}$$

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We are now ready to write our RR bulk action. In analogy with [11], we perform the field redefinition

$$\mathbf{A} = e^B\ \mathbf{C}\,, \tag{2.10}$$

which simplifies the calculations. As in [11], we group potential and field strengths into formal sums such as $\mathbf{A} = \sum_{p=odd} A^{(p)}$, $\mathbf{C} = \sum_{p=odd} C^{(p)}$, $\mathbf{G} = \sum_{p=even} G^{(p)}$. Equations involving these formal sums must be then considered term by term, projecting onto the forms of each given order p and considering only the relevant combinations. Similarly, we use the short-hand notation $e^B = 1 + B + (1/2)\, B \wedge B + \ldots$, and in expressions such as $e^B\,\mathbf{C}$ or $e^B\,\mathbf{G}$ the wedge product is understood. Then:

$$S_R + S_{CS} \longrightarrow S_{kin}(G) + S_{CS}(G,B) + S_{LM}(A,G)\,, \tag{2.11}$$

$$S_{kin}(G) \;=\; -\frac{1}{4}\int d^{10}x\ e_{10}\ \left[(G^{(0)})^2 + (G^{(2)})^2 + (G^{(4)})^2 + (G^{(6)})^2\right], \tag{2.12}$$

$$S_{LM}(A,G) \;=\; \frac{1}{2}\int \left(A^{(9)} - A^{(7)} + A^{(5)} - A^{(3)}\right) \wedge d(\mathbf{G}\ e^B)\,, \tag{2.13}$$

where sums restricted to the components in eq. (2.9) are implicit. In the part of the action containing the Lagrange multipliers, $S_{LM}(A,G)$, a sum over all the $G^{(p)}$ is understood, in order to fill the ten-dimensional wedge product. After the orbifold and orientifold projections, and with the choice of independent bulk fields specified in eq. (2.9), there is no surviving term from the ten-dimensional Chern-Simons action $S_{CS}(G,B)$ in eq. (2.11).

Finally, we can add contributions from D6-branes and O6-planes. Neglecting D-brane fluctuations and localized fluxes, they read

$$S_{D6/O6} = S_{DBI} + S_{WZ} = \sum_a^{D6,O6}\ \left[-T_a\int_{\pi_a} d^7x\ e_7\ e^{-\Phi} + \mu_a\int_{\pi_a} A^{(7)}\right], \tag{2.14}$$

where $T_a = \mu_a$ and branes/planes wrap factorisable 3-cycles $\pi_a = \prod_{A=1}^3 (n_A^a, m_A^a)$.

In summary, in the dual formulation the total action reads

$$S_{IIA}^{dual} = S_{NS} + S_{kin}(G) + S_{LM}(A,G) + S_{D6/O6}(A)\,, \tag{2.15}$$

with the four different contributions given by eqs. (2.2), (2.12), (2.13) and (2.14), respectively.

3. Bianchi identities and their solutions

Before discussing the BI for the RR sector, we first look at the BI for the NSNS 3-form, which reads:

$$dH = 0\,, \tag{3.1}$$

with general solution given by:

$$H = dB + \overline{H}\,, \qquad \overline{H} = \text{constant}\,. \tag{3.2}$$

In a trivial dimensional reduction, the first contribution gives the kinetic terms for the axions in B, and the second one is a constant flux. However, for generalized Scherk-Schwarz reductions (or twisted tori), characterized by constant parameters $\omega_{jk}{}^{i}$ (called $f^{i}{}_{kj}$ in [8] and C_{ijk} in [4]), the BI is modified as follows:

$$dH + \omega H = 0\,. \tag{3.3}$$

In the above equation and in the following, the transition from trivial to generalized dimensional reduction is described by the simple rule [8]:

$$\begin{aligned} dX &\rightarrow dX + \omega\ X\,, \\ \omega\ X &\equiv \omega_{\bullet\bullet}{}^{\times} X_{\times\bullet\cdots\bullet}\,, \end{aligned} \tag{3.4}$$

where X is a generic form and indices indicated with a full dot are antisymmetrized, while those with a cross are contracted. The constants ω (which may be called *geometrical fluxes*) can thus be viewed as operators that map a p-form into a $(p+1)$-form. According to the discussion of [8], appropriate for toroidal compactifications as we are considering here, the ω components must be structure constants of a Lie group, satisfying the Jacobi identity:

$$\omega\ \omega \equiv \omega_{\bullet\bullet}{}^{\times}\omega_{\times\bullet}{}^{\circ} = 0\,, \tag{3.5}$$

where an empty dot denotes a free index, and the additional condition

$$\omega_{\times\circ}{}^{\times} = 0 \tag{3.6}$$

must hold. While eq. (3.6) is automatically satisfied by the geometrical fluxes that survive our orientifold and orbifold projections, eq. (3.5) imposes a non-trivial constraint. As discussed in [9], this constraint can be interpreted as the BI for the non-trivial connection of the internal manifold, $d^2 = 0$, for constant geometrical fluxes ω. The general solution to eq. (3.3) is:

$$H = dB + \omega B + \overline{H}\,, \qquad \overline{H} = \text{constant}\,, \tag{3.7}$$

provided that:

$$\omega\ \overline{H} = 0\,, \tag{3.8}$$

which represents the integrability condition for the BI of H. In the class of compactifications under discussion, the solution for the surviving components of H reads:

$$\begin{aligned} (\mu\{56|78|910\}) &\ :\ H = dB\,, \\ (579|5810|6710|689) &\ :\ H = \omega\ B + \overline{H}\,, \end{aligned} \tag{3.9}$$

and eq. (3.8) is trivially satisfied because of the projections. As in [4], we have four NSNS fluxes $\overline{H}$ and twelve geometrical fluxes ω,

$$\begin{gathered} \omega_{68}{}^{10}\,, \qquad \omega_{106}{}^{8}\,, \qquad \omega_{810}{}^{6}\,, \\ \omega_{57}{}^{10}\,, \qquad \omega_{95}{}^{8}\,, \qquad \omega_{79}{}^{6}\,, \\ \omega_{58}{}^{9}\,, \qquad \omega_{89}{}^{5}\,, \qquad \omega_{67}{}^{9}\,, \qquad \omega_{96}{}^{7}\,, \qquad \omega_{105}{}^{7}\,, \qquad \omega_{710}{}^{5}\,, \end{gathered} \tag{3.10}$$

allowed by the orbifold and orientifold projections. However, as will be discussed in more detail in the next section, the constraints on these fluxes coming from the BI are not exactly the same as those derived in [4] from truncated $\mathcal{N}=4$ gaugings.

To derive the BI for the RR field strengths $G^{(p)}$ from the dual action (2.15), it is sufficient to vary it with respect to the corresponding Lagrange multipliers $A^{(9-p)}$. We can see that, if $A^{(9-p)}$ does not couple to localized sources, the general BI for $G^{(p)}$ read:

$$\begin{aligned} & d(e^B\ \mathbf{G}) = 0 \\ \Rightarrow\ & dG^{(p)} + H\wedge G^{(p-2)} = 0\,, \end{aligned} \tag{3.11}$$

which implies

$$dG^{(p)} + \omega\ G^{(p)} + H\wedge G^{(p-2)} = 0\,, \tag{3.12}$$

for generalized Scherk-Schwarz dimensional reduction. The general solution to equation (3.12) is:

$$G^{(p)} = dC^{(p-1)} + \omega\ C^{(p-1)} + H\wedge C^{(p-3)} + (\overline{\mathbf{G}}\ e^{-B})^{(p)}\,, \qquad (\overline{\mathbf{G}} = \text{constant})\,, \tag{3.13}$$

provided that the following integrability condition holds:

$$\omega\ \overline{G}^{(p)} + \overline{H}\wedge\overline{G}^{(p-2)} = 0\,. \tag{3.14}$$

The last term in eq. (3.13) is understood as expanded and projected into a p-form wedge product. The solution of eq. (3.13) is valid in general, and shows the power of the dual construction. Each p-form field strength in the action (2.12) receives contributions from a combination of all other fluxes with $p'\leq p$ and, as we will see in the next section, these mixing terms will be crucial to reconstruct the full $\mathcal{N}=1$ supersymmetric expression of the effective potential. In our case, the explicit solutions for the surviving components of the field strengths $G^{(p)}$ read:

$$\begin{aligned} & & G^{(0)} &= \overline{G}^{(0)}\,, \\ (56|\cdot|\cdot) &: & G^{(2)} &= \overline{G}^{(2)} - B\,\overline{G}^{(0)}\,, \\ (5678|\cdot|\cdot) &: & G^{(4)} &= \omega\, C^{(3)} + \overline{G}^{(4)} - B\wedge\overline{G}^{(2)} + \frac{1}{2}B\wedge B\,\overline{G}^{(0)}\,, \\ (\mu\{6810|\cdot|\cdot|\cdot\}) &: & G^{(4)} &= dC^{(3)}\,, \\ (5678910) &: & G^{(6)} &= \overline{H}\wedge C^{(3)} + (\omega\, B)\wedge C^{(3)} + \overline{G}^{(6)} \\ & & & \quad -B\wedge\overline{G}^{(4)} + \frac{1}{2}B\wedge B\wedge\overline{G}^{(2)} - \frac{1}{6}B\wedge B\wedge B\,\overline{G}^{(0)}\,, \end{aligned} \tag{3.15}$$

while the corresponding constraints of eq. (3.14) are all trivially satisfied after the projections.

Finally, we need to discuss separately the case of the *phantom* components of $G^{(2)}$, whose dual Lagrange multipliers $A^{(7)}$ couple to D6-branes and O6-planes. The corresponding BI read:

$$\frac{1}{2}\left(dG^{(2)} + \omega\, G^{(2)} + HG^{(0)}\right) = \sum_{a}^{D6,O6}\ \mu_a \delta^{(3)}(\pi_a)\,, \tag{3.16}$$

where $\delta^{(3)}(\pi_a)$ is the 3-form Poincaré-dual to the 3-cycle π_a of the brane/plane a. We are not interested in solving this equation, since the solution for $G^{(2)}$ is odd under the combined orientifold and orbifold projections and does not enter the effective potential. However, the integrability conditions associated to the above BI give, in components, the following non-trivial constraints:

$$\begin{aligned}
\frac{1}{2}\left(\omega\,\overline{G}^{(2)}+\overline{H}\,\overline{G}^{(0)}\right)_{579} &= \sum_a^{D6,O6}\mu_a m_1^a m_2^a m_3^a\,,\\
\frac{1}{2}\left(\omega\,\overline{G}^{(2)}+\overline{H}\,\overline{G}^{(0)}\right)_{5810} &= \sum_a^{D6,O6}\mu_a m_1^a n_2^a n_3^a\,,\\
\frac{1}{2}\left(\omega\,\overline{G}^{(2)}+\overline{H}\,\overline{G}^{(0)}\right)_{6710} &= \sum_a^{D6,O6}\mu_a n_1^a m_2^a n_3^a\,,\\
\frac{1}{2}\left(\omega\,\overline{G}^{(2)}+\overline{H}\,\overline{G}^{(0)}\right)_{689} &= \sum_a^{D6,O6}\mu_a n_1^a n_2^a m_3^a\,,
\end{aligned}\tag{3.17}$$

where (n_A^a, m_A^a) are the wrapping numbers of the brane/plane a on the A-th 2-torus.

These constraints show how the RR-tadpole cancellation conditions for D6-branes are modified in the presence of general fluxes. It is interesting to notice that there exist combinations of fluxes that escape the usual charge cancellation conditions for D6-branes and O6-planes on each 3-cycle. This is analogous to what happens in type-IIB compactifications with both RR and NSNS 3-form fluxes turned on [15]. In that case, however, the contribution of fluxes to both the scalar potential and the BI is either vanishing or positive-definite, leading to no-scale models or runaway potentials[3]. On the other hand, in type-IIA compactifications the richness of the flux structure allows us to avoid such bounds, enlarging the possibilities for moduli stabilization and model building. It would be also interesting to see whether these modifications of the BI can admit stable supersymmetric configurations of intersecting D6-branes on toroidal compactifications without orbifolds.

4. $\mathcal{N}=1$ effective potential and superpotential

We can now derive the effective $D=4$ action for the bulk scalar fields of our theory by substituting the solutions of the BI, as found in the previous section, into the action of eq. (2.15). The kinetic terms are fixed by the Kähler potential[4]

$$K=-\log Y\,,\qquad Y=s\,t_1\,t_2\,t_3\,u_1\,u_2\,u_3\,,\tag{4.1}$$

where the seven complex scalars

$$S=s+i\sigma\,,\qquad T_A=t_A+i\tau_A\,,\qquad U_A=u_A+i\nu_A\,,\qquad (A=1,2,3)\,,\tag{4.2}$$

[3] Strictly speaking, this is true for the Calabi-Yau compactifications considered in [15] and for the one considered here (i.e. $T^6/Z_2\times Z_2$), but other possibilities can be studied (for a recent example, see e.g. [16]).

[4] Notice that our conventions for Y differ from those of [4] by a constant multiplicative factor 2^7, which can be reabsorbed as an overall multiplicative factor in the superpotential W.

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are connected to the ten-dimensional fields via:

$$g_{MN} = \text{blockdiag}\left(\hat{s}^{-1}\,\widetilde{g}_{\mu\nu},\ t_1\,\hat{u}_1,\ \frac{t_1}{\hat{u}_1},\ t_2\,\hat{u}_2,\ \frac{t_2}{\hat{u}_2},\ t_3\,\hat{u}_3,\ \frac{t_3}{\hat{u}_3}\right), \tag{4.3}$$

$$e^{-2\Phi} = \frac{\hat{s}}{t_1 t_2 t_3}\,,\quad B_{56|78|910} = \tau_{1|2|3}\,,\quad C^{(3)}_{6810} = \sigma\,,\quad C^{(3)}_{679|589|5710} = -\nu_{1|2|3}\,,$$

$$s = \sqrt{\frac{\hat{s}}{\hat{u}_1\,\hat{u}_2\,\hat{u}_3}}\,,\qquad u_1 = \sqrt{\frac{\hat{s}\ \hat{u}_2\,\hat{u}_3}{\hat{u}_1}}\,,\quad u_2 = \sqrt{\frac{\hat{s}\ \hat{u}_1\,\hat{u}_3}{\hat{u}_2}}\,,\qquad u_3 = \sqrt{\frac{\hat{s}\ \hat{u}_1\,\hat{u}_2}{\hat{u}_3}}\,, \tag{4.4}$$

and $\widetilde{g}_{\mu\nu}$ is the metric in the $D = 4$ Einstein frame. In this paper, we call *main* moduli the seven complex scalars in (4.2), *geometrical* moduli their real parts, *axions* their imaginary parts.

The $D = 4$ scalar potential receives contributions from the Einstein term, the generalized kinetic term containing the NSNS three form, the analogous terms for the RR field strengths, and finally the tensions of localized objects:

$$V = V_E + V_H + V_G + V_6\,. \tag{4.5}$$

The first contribution is the well-known Scherk-Schwarz potential [8]:

$$V_E = \frac{1}{8}\frac{e_{10}}{\widetilde{e}_4}e^{-2\Phi}\left(\omega_{jk}{}^{i}\,\omega_{j'k'}{}^{i'}\ g_{ii'}\ g^{jj'}\ g^{kk'} + 2\,\omega_{jk}{}^{i}\ \omega_{j'i}{}^{k}\ g^{jj'}\right), \tag{4.6}$$

which involves only the geometrical fluxes ω. The second contribution comes from the $(H)^2$ term in eq. (2.2):

$$V_H = \frac{1}{4}\frac{e_{10}}{\widetilde{e}_4}e^{-2\Phi}\left(\overline{H} + \omega\,B\right)^2\,. \tag{4.7}$$

Then we have the contributions coming from S_{kin} of the RR sector:

$$\begin{aligned} V_G = \frac{1}{4}\frac{e_{10}}{\widetilde{e}_4}\Bigg[&\left(\overline{G}^{(0)}\right)^2 + \left(\overline{G}^{(2)} - B\overline{G}^{(0)}\right)^2 + \left(\omega\,C^{(3)} + \overline{G}^{(4)} - B\wedge\overline{G}^{(2)} + \frac{1}{2}B\wedge B\,\overline{G}^{(0)}\right)^2 + \\ &+\Big(\overline{H}\wedge C^{(3)} + (\omega\,B)\wedge C^{(3)} + \overline{G}^{(6)} - B\wedge\overline{G}^{(4)} + \\ &+\frac{1}{2}B\wedge B\wedge\overline{G}^{(2)} - \frac{1}{6}B\wedge B\wedge B\,\overline{G}^{(0)}\Big)^2\Bigg]. \end{aligned} \tag{4.8}$$

Finally, we have the contributions coming from D6-branes and O6-planes:

$$V_{D6/O6} = \sum_a T_a \frac{e_7^{(a)}}{\widetilde{e}_4}e^{-\Phi}\,, \tag{4.9}$$

where $e_7^{(a)}$ refers to the induced metric on the D6-brane/O6-plane under consideration. The potential $V_{D6/O6}$ coming from localized sources can be conveniently rewritten by exploiting the requirement that the brane setup preserves $\mathcal{N} = 1$ supersymmetry. In this case, in fact:

$$V_{D6/O6} = e^K t_1 t_2 t_3 \sum_a T_a\left(m_1^a m_2^a m_3^a\ s - m_1^a n_2^a n_3^a\ u_1 - n_1^a m_2^a n_3^a\ u_2 - n_1^a n_2^a m_3^a\ u_3\right), \tag{4.10}$$

where, recalling that $\mu_a = T_a$ because of supersymmetry, we find the same combination of wrapping numbers present in the BI of eq. (3.17). We can then rewrite the localized contributions to the potential as:

$$\begin{aligned} V_{D6/O6} = \frac{1}{2} e^K t_1 t_2 t_3 \Big[& (\omega \overline{G}^{(2)} + \overline{H}\, \overline{G}^{(0)})_{579}\ s - (\omega \overline{G}^{(2)} + \overline{H}\, \overline{G}^{(0)})_{5810}\ u_1 \\ & - (\omega \overline{G}^{(2)} + \overline{H}\, \overline{G}^{(0)})_{6710}\ u_2 - (\omega \overline{G}^{(2)} + \overline{H}\, \overline{G}^{(0)})_{689}\ u_3 \Big], \end{aligned} \tag{4.11}$$

which is *independent* of the particular setup of D6-branes and O6-planes, as long as they preserve $\mathcal{N} = 1$. Their contribution to the potential enters only indirectly, via the constraints on the fluxes spelled out in eq. (3.17). We verified that the terms in eq. (4.11) are precisely those that need to be added to the bulk potential, $V_E + V_H + V_G$, to reconstruct the standard $\mathcal{N} = 1$ formula:

$$V = e^K \left[\sum_{\alpha=1}^{7} |W_\alpha(\varphi) + K_\alpha W(\varphi)|^2 - 3|W(\varphi)|^2 \right], \qquad \varphi^{1,\dots,7} = (S, T_1, T_2, T_3, U_1, U_2, U_3)\,, \tag{4.12}$$

where $W_\alpha \equiv \partial W / \partial \varphi^\alpha$, $K_\alpha \equiv \partial K / \partial \varphi^\alpha$, with superpotential

$$\begin{aligned} 4\,W = & \left(\omega_{810}{}^{6}\, T_1\, U_1 + \omega_{106}{}^{8}\, T_2\, U_2 + \omega_{68}{}^{10}\, T_3\, U_3\right) - S\left(\omega_{79}{}^{6}\, T_1 + \omega_{95}{}^{8}\, T_2 + \omega_{57}{}^{10}\, T_3\right) - \\ & -\left(\omega_{89}{}^{5} T_1 U_3 + \omega_{96}{}^{7}\, T_2\, U_3 + \omega_{710}{}^{5} T_1\, U_2 + \omega_{67}{}^{9} T_3\, U_2 + \omega_{105}{}^{7} T_2 U_1 + \omega_{58}{}^{9} T_3 U_1\right) + \\ & + i\left(\overline{G}^{(4)}_{78910}\, T_1 + \overline{G}^{(4)}_{91056}\, T_2 + \overline{G}^{(4)}_{5678}\, T_3\right) - \left(\overline{G}^{(2)}_{56}\, T_2\, T_3 + \overline{G}^{(2)}_{78}\, T_1\, T_3 + \overline{G}^{(2)}_{910}\, T_1\, T_2\right) + \\ & + \overline{G}^{(6)} - i\, \overline{G}^{(0)}\, T_1\, T_2\, T_3 + i\left(\overline{H}_{579}\, S - \overline{H}_{5810}\, U_1 - \overline{H}_{6710}\, U_2 - \overline{H}_{689}\, U_3\right), \end{aligned} \tag{4.13}$$

provided that eq. (3.5) is satisfied. We stress the importance of the mixing terms coming from the solutions of the BI in eq. (3.15) for recovering the entire potential of eq. (4.12) by dimensional reduction.

The general superpotential of eq. (4.13) reduces to the one found in [4], under the simplifying assumption of a symmetry with respect to the interchange of the three factorized two-tori (plane-interchange-symmetry). It also takes a suggestive and compact form when rewritten in terms of geometrical objects:

$$W = \frac{1}{4} \int_{\mathcal{M}_6} \overline{\mathbf{G}}\, e^{i J^c} - i\, \left(\overline{H} - i\omega\, J^c\right) \wedge \Omega^c\,, \tag{4.14}$$

where, in our conventions, J^c and Ω^c read

$$\begin{aligned} J^c &= J + i\, B\,, & J &= \frac{i}{2} \sum_{A=1}^{3} dz^A \wedge d\overline{z}^A\,, \\ \Omega^c &= \mathrm{Re}\left(i\, e^{-\Phi}\, \Omega\right) + i\, C^{(3)}\,, & \Omega &= dz^1 \wedge dz^2 \wedge dz^3\,. \end{aligned} \tag{4.15}$$

In particular, in our case:

$$J^c_{56|78|910} = T_{1|2|3}\,, \tag{4.16}$$

$$\Omega^c_{6810} = S\,, \qquad \Omega^c_{679|589|5710} = -U_{1|2|3}\,. \tag{4.17}$$

Since $\mathrm{Re}(iJ^c) = -B$, we can recognize in eq. (4.14) the combinations of fluxes and fields entering the BI of eqs. (3.9) and (3.13). Indeed:

$$W = \frac{1}{4}\int_{\mathcal{M}_6} \widetilde{G}^{(6)}\,, \tag{4.18}$$

where $\widetilde{G}^{(6)}$ is the solution $G^{(6)}$ of the BI in eq. (3.15), with the substitutions:

$$B \to -iJ^c\,, \qquad C^{(3)} \to -i\;\Omega^c\,. \tag{4.19}$$

Moreover, since by T-duality:

$$\overline{\mathbf{G}}\; e^{iJ^c} \leftrightarrow \overline{G}^{(3)} \wedge \Omega_{IIB}\,, \tag{4.20}$$

eq. (4.14) exhibits the same structure of the Gukov-Vafa-Witten [17] IIB superpotential:

$$\int_{\mathcal{M}_6} \left(\overline{G}^{(3)} - iS\overline{H}\right) \wedge \Omega_{IIB}\,, \tag{4.21}$$

apart from the twisted tori contribution proportional to $\omega\, J^c \wedge \Omega^c = dJ^c \wedge \Omega^c$. Parts of the superpotential of eq. (4.14) have already been argued in other contexts by using mirror symmetries or dimensional reduction with torsion (see e.g. [12−14]). All this makes us think that the validity of eq. (4.14) extends beyond the $T^6/(Z_2 \times Z_2)$ orbifold considered here, although we explicitly derived it only for this case.

The explicit presence of all the Kähler (J^c), complex structure and dilaton (Ω^c) moduli in the IIA superpotential of eq. (4.14) explains why in this theory it is possible to find stable vacua, with all geometrical moduli fixed, in the perturbative regime, and makes type-IIA compactifications appealing candidates for perturbatively addressing the problem of moduli stabilization.

4.1 Relation with $\mathcal{N} = 4$ gaugings

We can now study the relations between the conditions found in [4] by requiring that the effective field theory be a truncation of a gauged $\mathcal{N} = 4$ supergravity, and those found here by dimensional reduction and BI, requiring only $\mathcal{N} = 1$ supersymmetry.

Reference [9] found that in the heterotic case there is a one-to-one correspondence between Jacobi identities of the $\mathcal{N} = 4$ gauged supergravity and Bianchi identities in the presence of general fluxes. As we will see in a moment, this is no longer true, in general, for type-II compactifications. In [4], the formalism of gauged $\mathcal{N} = 4$ supergravity was used to derive, upon consistent truncation, the superpotential of eq. (4.13). That work also found the consistency conditions on the fluxes implied by the closure of the Jacobi identities of the gauged $\mathcal{N} = 4$, $D = 4$ Lie algebra, at the compensator level. In [10], the analysis of [9] was extended to M-theory and to more general type-II compactifications. From these analyses, we can conclude that the constraints coming from $\mathcal{N} = 4$ supergravity can be divided into two groups. The first group contains some of the equations in (3.5), while the second group is made of the last three BI of eq. (3.17) with vanishing r.h.s. In contrast with the heterotic case, the conditions for a consistent $\mathcal{N} = 4$ $D = 4$ gauging do not seem sufficient to reconstruct all the consistency conditions coming from toroidal generalized dimensional reductions, in particular the closure of all the Scherk-Schwarz BI (3.5). It is not known if these type of $\mathcal{N}=4$ gaugings can have a geometrical interpretation. On the other hand, the conditions from the second group are more restrictive than those found here from generalized $\mathcal{N}=1$ dimensional reduction. In this case, however, we can give a definite interpretation: since different branes/planes preserve different $\mathcal{N}=4$ subsets of the $\mathcal{N}=8$ bulk supersymmetry, depending on their orientation, the vanishing of the r.h.s. in the last three BI of eq. (3.17) corresponds to requiring that all the D6-branes preserve the same $\mathcal{N}=4$. If this condition is not satisfied, the effective field theory cannot be seen as a simple truncation of $\mathcal{N}=4$ gauged supergravity. It is worth noticing, however, that the $\mathcal{N}=1$ formalism of [4] is able to reproduce the correct form of the effective superpotential even for general brane configurations, at least for the bulk fields. Consistency conditions for the fluxes, on the other hand, require the derivation of the BI from reduction, analogously to what has been done in the present work.

5. Examples of consistent $\mathcal{N}=1$ vacua

In reference [4], a stable supersymmetric AdS_4 vacuum (satisfying the criteria[5] of [18]) was constructed, starting from the superpotential of eq. (4.13), in the plane-interchange-symmetric limit. It was found that for the following choice of fluxes:

$$\begin{aligned} -\frac{1}{9}\overline{G}^{(6)} &= \overline{G}^{(2)} = \frac{1}{6}\omega_1 = -\frac{1}{2}\omega_2\,, \\ \frac{1}{3}\overline{G}^{(4)} &= \frac{1}{5}\overline{G}^{(0)} = -\frac{1}{2}\overline{H}_0 = \frac{1}{2}\overline{H}_1\,, \\ \omega_3 &= 0\,, \end{aligned} \tag{5.1}$$

where, in our notation:

$$\begin{aligned} \omega_1 &\equiv \omega_{68}{}^{10} = \omega_{106}{}^{8} = \omega_{810}{}^{6}\,, \\ \omega_2 &\equiv \omega_{57}{}^{10} = \omega_{95}{}^{8} = \omega_{79}{}^{6}\,, \\ \omega_3 &\equiv \omega_{58}{}^{9} = \omega_{89}{}^{5} = \omega_{67}{}^{9} = \omega_{96}{}^{7} = \omega_{105}{}^{7} = \omega_{710}{}^{5}\,, \\ \overline{H}_0 &\equiv H_{579}\,, \\ \overline{H}_1 &\equiv H_{5810|6710|689}\,, \end{aligned} \tag{5.2}$$

there exists a stable supersymmetric AdS_4 vacuum at $\langle S\rangle = \langle T_A\rangle = \langle U_A\rangle = 1$, with $\langle V\rangle = -3\langle e^K|W|^2\rangle$, such that all seven geometrical moduli are fixed. As explained in [4], the quantization condition on the fluxes can be made manifest by a suitable rescaling of the moduli. Moreover, the fluxes in eq. (5.1) automatically satisfy the $\mathcal{N}=4$ constraints:

$$\omega_3\,(\omega_3-\omega_1) = 0\,, \qquad 5\,\overline{H}_1^2 = 3\,\omega_2^2\,. \tag{5.3}$$

[5]Notice that requiring a stable supersymmetric AdS_4 vacuum does not imply that all the background values of the moduli are fixed.

The first one corresponds to one of the geometrical BI of eq. (3.5). The second one represents the absence of localized RR sources from 3-cycles non-parallel to the $\mathcal{N}=4$ O6-plane (6,8,10). However, for the vacuum solution to be fully consistent with a Scherk-Schwarz reduction, also the following constraint coming from the geometrical BI (3.5) must be satisfied:

$$\omega_2\,(\omega_1-\omega_3)=0\,. \tag{5.4}$$

It is not difficult to find solutions that extend eq. (5.1) and also satisfy eq. (5.4). If the fluxes are chosen so that:

$$\frac{1}{9}\overline{G}^{(6)} = -t_0^2\,\overline{G}^{(2)} = \frac{t_0\,u_0}{6}\omega_1 = \frac{s_0\,t_0}{2}\omega_2 = \frac{t_0\,u_0}{6}\omega_3\,, \tag{5.5}$$

$$\frac{t_0}{3}\overline{G}^{(4)} = \frac{t_0^3}{5}\overline{G}^{(0)} = -\frac{s_0}{2}\overline{H}_0 = \frac{u_0}{2}\overline{H}_1\,, \tag{5.6}$$

all the BI are satisfied and there is a supersymmetric AdS_4 vacuum at $\langle S\rangle = s_0$, $\langle T_A\rangle = t_0$ and $\langle U_A\rangle = u_0$ ($s_0\,,\,t_0\,,\,u_0\in\mathbb{R}^+$). All seven main moduli are stabilized in the sense of [18]. Moreover, all seven geometrical moduli are fixed and, for generic values of the two independent parameters in eqs. (5.5) and (5.6), also all axions are fixed, with the exception of a single residual Goldstone mode, corresponding to a combination of $(\sigma,\nu_1,\nu_2,\nu_3)$. The constraints of eqs. (5.5) and (5.6) represent a family of supersymmetric AdS_4 solutions of the model compatible with the Scherk-Schwarz reduction. Other solutions can be obtained by performing "electric" duality transformations such as axionic shifts or dilatations, and by relaxing the constraints from plane-interchange-symmetry.

The second relation of eq. (5.3) reads now

$$5\,u_0^2\,\overline{H}_1 = 3\,s_0^2\,t_0^2\,\omega_2^2\,. \tag{5.7}$$

When it is satisfied, the AdS_4 vacua of eq. (5.6) also admit an interpretation in terms of $\mathcal{N}=4$ gaugings. However, the generalized $\mathcal{N}=1$ reduction performed in the present paper allows us to study a wider class of solutions that do not admit such an interpretation. For example, we could choose either $\overline{H}_1=0$ or $\omega_2=0$, and compensate for the mismatch of RR charges with suitable D6-brane configurations. Summarizing, we can construct stable $\mathcal{N}=1$ AdS_4 vacua by switching on either RR and geometrical fluxes ($\overline{G}^{(6)}$, $\overline{G}^{(2)}$ and $\omega_{1,2,3}$), or RR and NSNS fluxes ($\overline{G}^{(4)}$, $\overline{G}^{(0)}$ and $\overline{H}_{0,1}$), or both. However, for the solution to admit a $\mathcal{N}=4$ gauging interpretation, all the possible types of fluxes have to be turned on simultaneously.

Vacua corresponding to no-scale models or to runaway potentials can be easily found as well, analogously to what was done in [4]. Also in these cases, however, there is not a one-to-one correspondence between the solutions compatible with generalized $\mathcal{N}=1$ dimensional reduction and those satisfying the $\mathcal{N}=4$ gauging conditions of the $D=4$ effective field theory.

6. Conclusions and outlook

In this paper we considered $\mathcal{N}=1$ compactifications of type-IIA supergravity on the O6 orientifold, consistently combined with the $T^6/(Z_2\times Z_2)$ orbifold, in the presence of D6-

branes and of general NSNS, RR and Scherk-Schwarz geometrical fluxes. We introduced a suitable dual action, to derive and solve the BI, and to compute, by $\mathcal{N} = 1$ generalized dimensional reduction, the effective scalar potential and superpotential. We derived an elegant geometrical expression for the superpotential, which generalizes previous results. We also identified a family of fluxes that are compatible with all the BI, including the geometrical ones, and perturbatively fix all seven geometrical moduli of the closed string sector on a stable supersymmetric AdS_4 vacuum. We finally clarified the relation of this approach with the one of [4], based on the constraints coming from $\mathcal{N} = 4$ gaugings.

Among the possible extensions of the present work, needed to proceed towards realistic models, the most immediate ones seem to be the inclusion of brane fields, the inclusion of localized magnetic fluxes and the calculation of soft terms (along the lines of [19, 20]).

Another open question, which may be more difficult to address, is the extension of our results to compactifications with non-negligible warping.

Acknowledgments

We thank G. Dall'Agata, J.-P. Derendinger, S. Ferrara and C. Kounnas for discussions. This work was supported in part by the European Programme "The Quest For Unification", contract MRTN-CT-2004-503369.

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